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Modern Decision Methodology in Radiochemical Testing

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Decision methodology

Decision methodology in measurements of analytical quantities addresses:

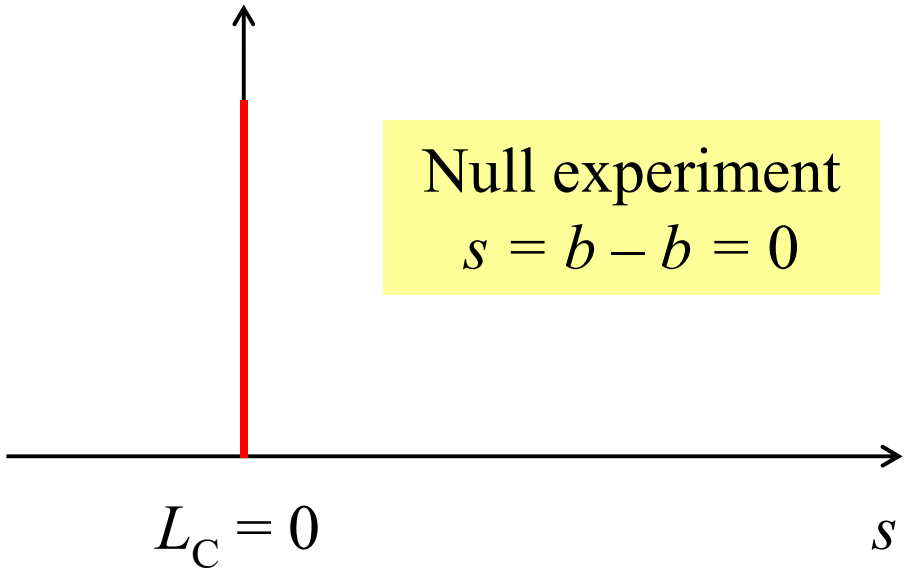
i. the measures of detectability
ii. the decision whether a detected quantity is distinguishable from the noise
iii. if the detected result meets data objectives

Classical decision methodology

$$s = c - b$$

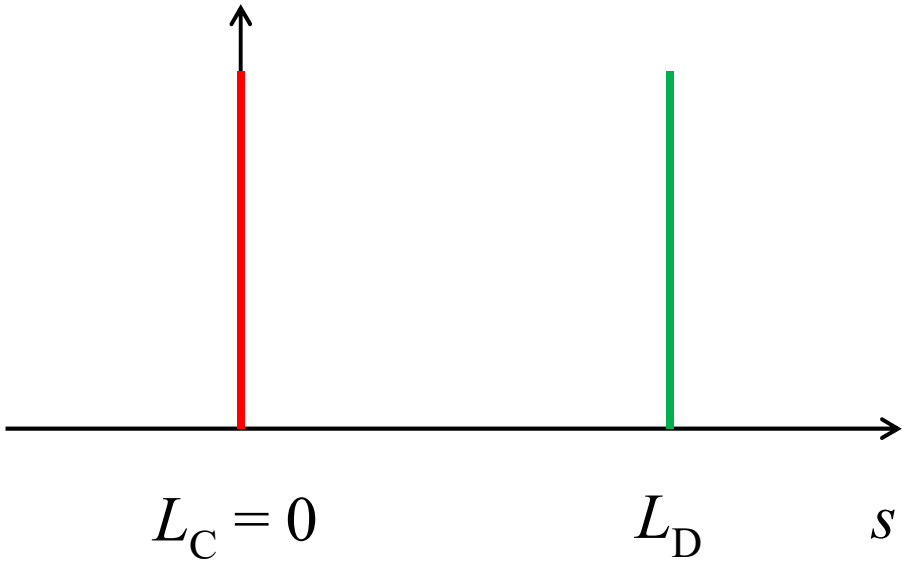
c	gross sample signal (counts), measured
b	background signal (counts), measured separately
s	net sample signal (counts), would like to have

Hypothetical case: no statistical fluctuations



$s = L_C$, not detected
$s > L_C$, detected

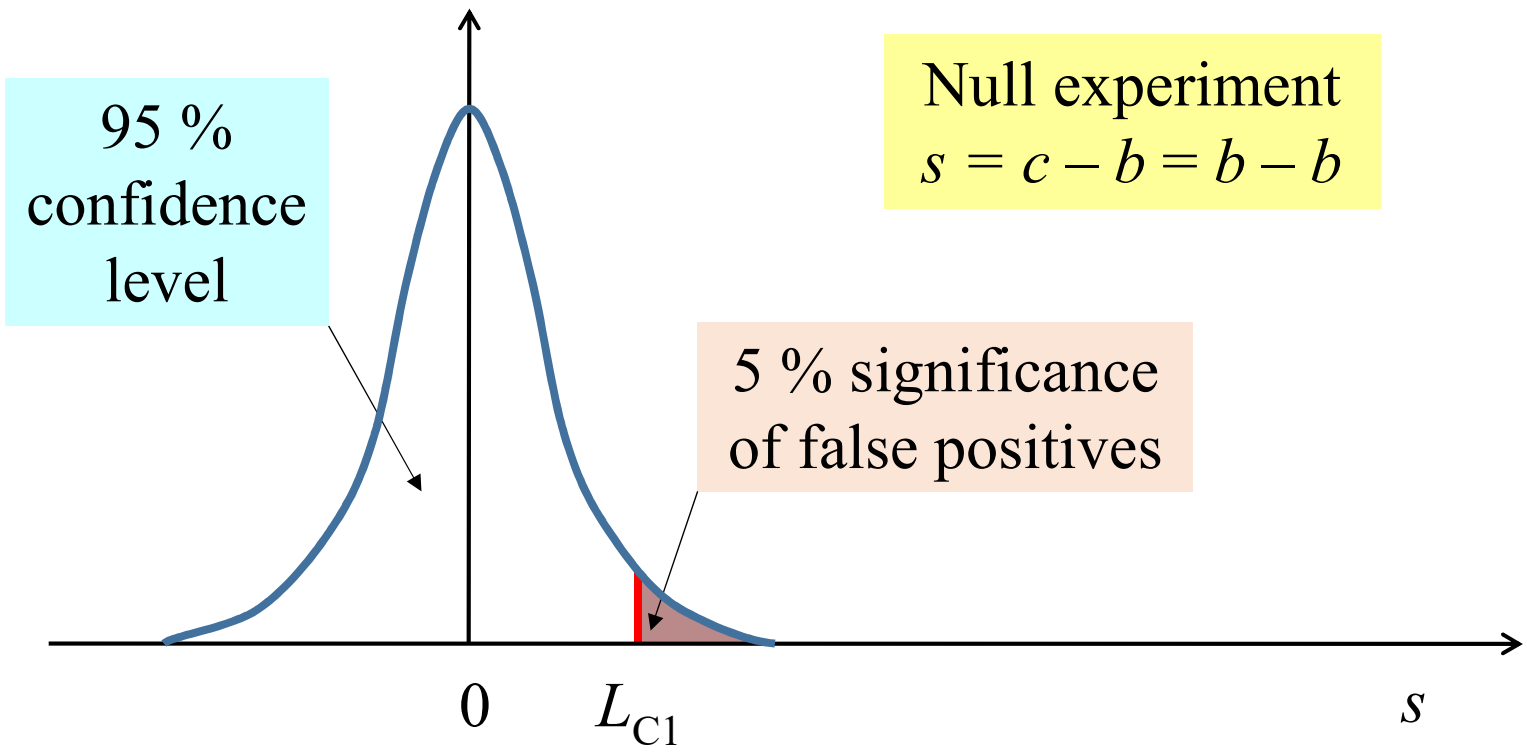
L_C is decision level above the noise



$s = L_C$,	not detected
$L_C < s \leq L_D$,	no decision
$s > L_D$,	detected

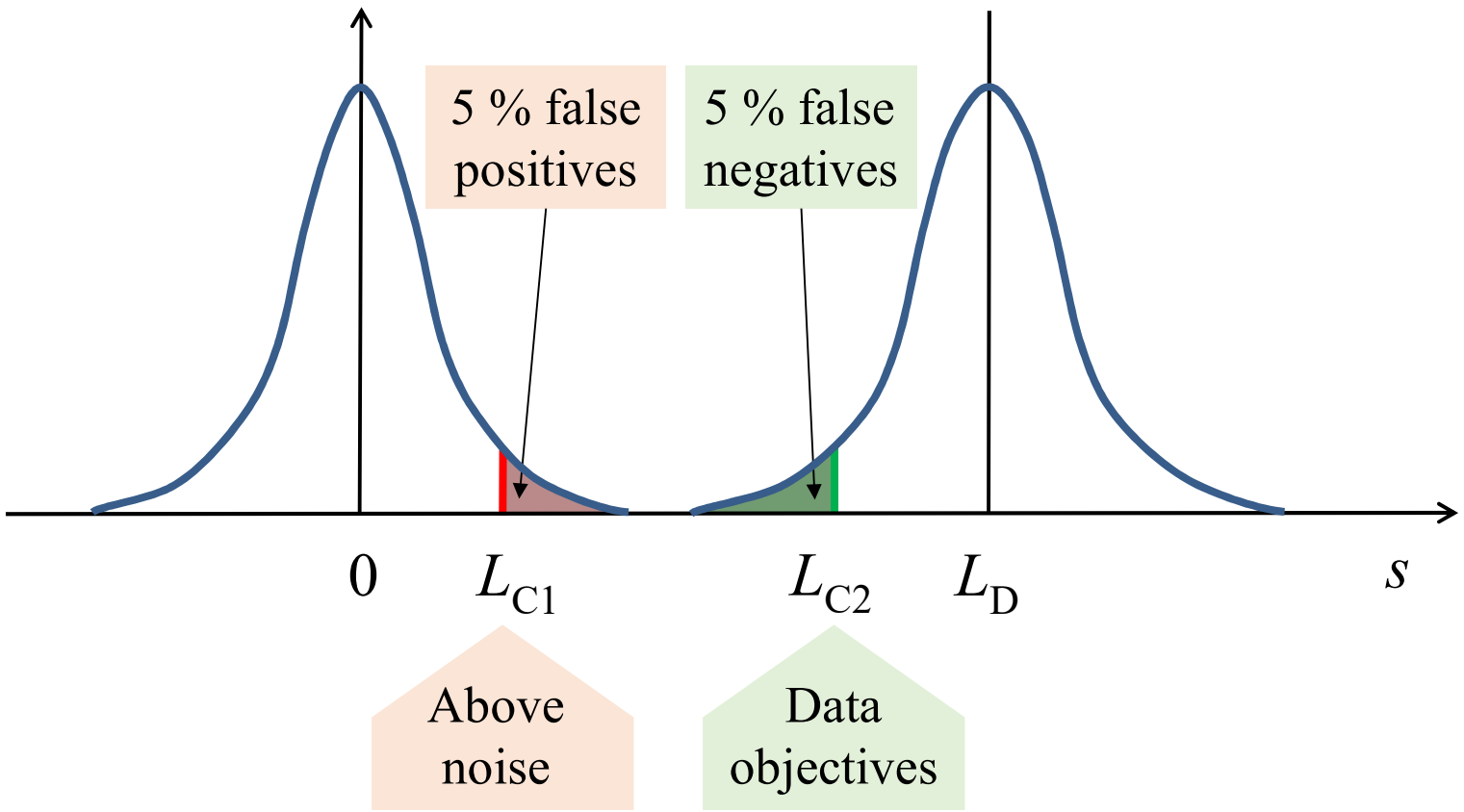
L_D is detection limit which meets data objectives

Including statistical fluctuations



$s \leq L_{C1}$, not detected

$s > L_{C1}$, detected



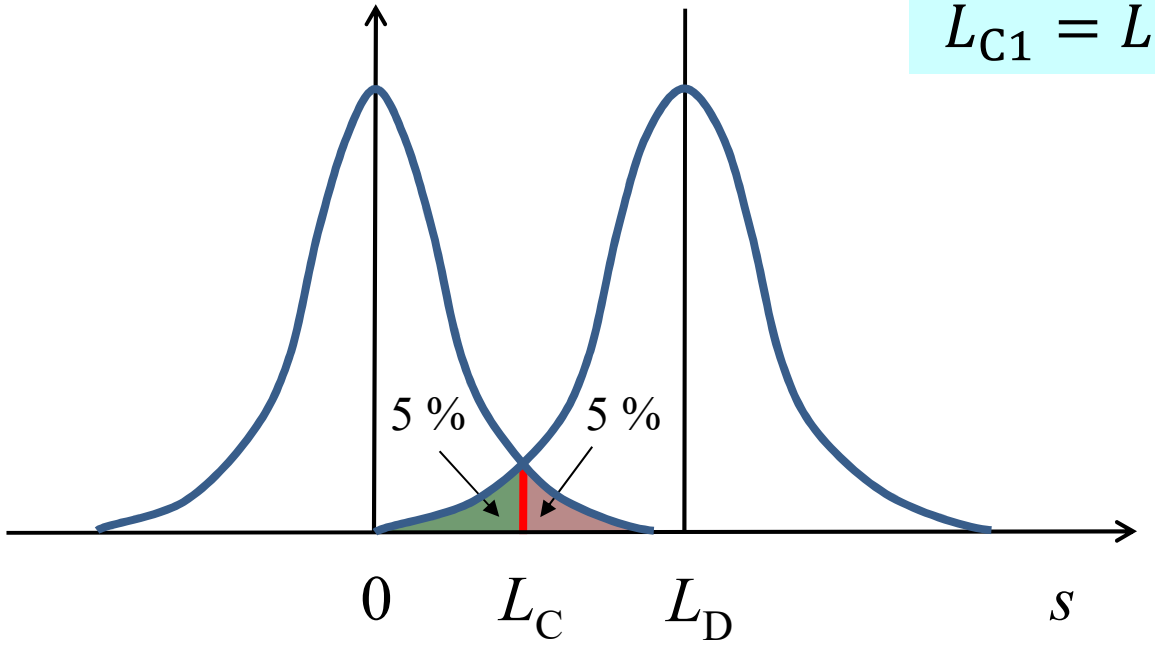
$s \leq L_{C1}$, not detected

$L_{C1} < s \leq L_{C2}$, no decision

$s > L_{C2}$, detected

Special case: “no decision” region eliminated

$$L_{C1} = L_{C2} = L_C$$

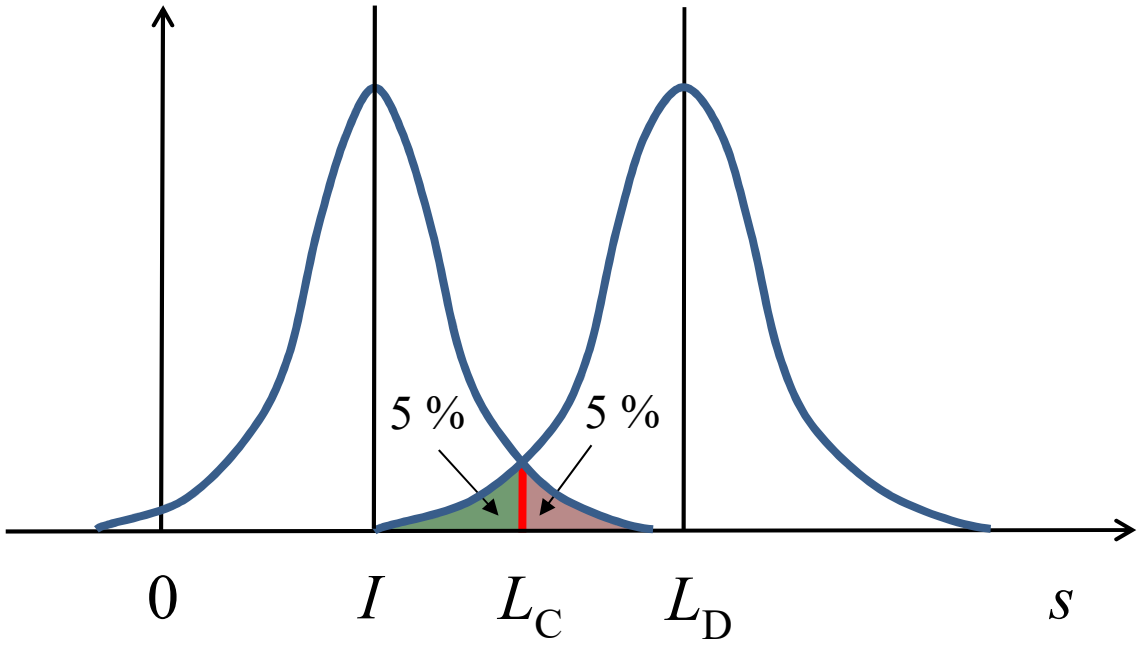


$s \leq L_C$, not detected

$s > L_C$, detected

$L_D \equiv$ MDA, minimum detectable activity, derivable in radiochemistry

Average of null signal > 0: interference or positive MB



Report as is, even if negative, or censor ?

Quantity	Case result < 0	Case result > 0
Result	–1.0	1.0
Sigma	1.0	1.0
Report as is	–1.0 ± 1.0	1.0 ± 1.0
Decision level	1.6·1.0 = 1.6	1.6·1.0 = 1.6
Decision	not detected	not detected
Upper limit	–1.0 + 1.6 = 0.6	1.0 + 1.6 = 2.6
Censor	< 0.6	< 2.6

Report	Advantage	Disadvantage
As is	possible to average	can be a fluke
Censor	risk assessment	not possible to average

Appendix

Selected literature

- Altshuler B., Pasternack B. (1963). Statistical measures of the lower limit of detection of a radioactivity counter. *Health Physics* **9** 293-298.
- Currie L.A. (1968). Limits for qualitative detection and quantitative determination, application to radiochemistry. *Analytical Chemistry* **40** 586-593.
- Gogolak C.V., Powers G.E., Huffert A.M. (1998). A nonparametric statistical methodology for the design and analysis of final status decommissioning surveys. U.S. Nuclear Regulatory Commission. *Report* NUREG-1505, Rev. 1.
- Multi-Agency Radiological Laboratory Analytical Protocols Manual (MARLAP, 2004), Volume III, Chapter 20. *Report* NUREG-1576, EPA 402-B-04-001C, NTIS PB2004-105421.
- Lyons L. (2012). Discovery or fluke: statistics in particle physics. *Physics Today* **65**(7) 45-51.
- Semkow T.M., Beach S.E., Khan A.J., Bari A., Bradt C.J., Haines D.K., Syed U.-F. (2012). Multi-window counting of radioactivity. *Nuclear Instruments and Methods in Physics Research A* **664** 236-244.

Numerical example

Detector efficiency	Sample mass (kg)	1-Sided CL	k_1	2-Sided CL	k_2
0.10	0.60	0.95	1.64	0.95	1.96
Quantity	Background	Cs-137 source	Sample 1	Sample 2	Sample 3
Time (s)	1000	10	100	100	100
Counts	4888	2841	502	466	531
Net rate (c/s)		279.21	0.13	– 0.23	0.42
Decision level (c/s)		1.21	0.39	0.39	0.39
Detected?		Yes	No	No	Yes
Upper limit (c/s)			0.52	0.15	
Uncertainty of net rate (c/s)		10.54			0.64
Detection limit (c/s)		2.70	0.80	0.80	0.80
Activity (Bq/kg)		4653.53			7.03
Activity uncertainty (Bq/kg)		174.13			7.87
Upper limit (Bq/kg)			8.73	2.51	
MDA (Bq/kg)		44.94	13.36	13.36	13.36

Derivations for the numerical example in Table

L_C	decision level
L_D	detection limit
L_U	upper limit
c	gross sample signal (counts)
b	background signal (counts)
t_s	gross sample counting time
t_b	background counting time
R_c	gross sample counting rate
R_b	background counting rate
R_s	net sample counting rate
σ^2	variance
r	Gaussian continuity correction
k_1	1-sided deviate (1.645 for 95 % confidence level)
k_2	2-sided deviate (1.960 for 95 % confidence level)
ε	detection efficiency
m	sample mass
V	sample volume

$$R_c = \frac{c}{t_s}\,, \qquad R_b = \frac{b}{t_b}\,, \qquad R_s = R_c - R_b$$

$$\sigma^2(R_s) = \sigma^2(R_c) + \sigma^2(R_b) = \frac{R_c}{t_s} + \frac{R_b}{t_b} = \frac{R_s}{t_s} + R_b\left(\frac{1}{t_s} + \frac{1}{t_b}\right)$$

$$\sigma^2(R_s = 0) = 2rR_b \qquad r = \frac{1}{2}\left(\frac{1}{t_s} + \frac{1}{t_b}\right)$$

$$L_C = r + k_1\sigma(R_s = 0)$$

$$L_U = r + R_s + k_1\sigma(R_s)$$

decision rule:

$R_s \le L_C$, not detected

$R_s > L_C$, detected

if detected, report	$R_s \pm k_2\sigma(R_s)$
if not detected, report	L_U

$$L_D = r + L_C + k_1\sigma(R_S = L_D) = r + L_{\mathrm{C}} + \frac{k_1^2}{t_{\mathrm{S}}} + \sqrt{\left(r + L_{\mathrm{C}} + \frac{k_1^2}{t_{\mathrm{S}}}\right)^2 - 4rL_{\mathrm{C}}}$$

$$\cong 2L_{\mathrm{C}} + \frac{k_1^2}{t_{\mathrm{S}}} + 2r\frac{2t_{\mathrm{S}}r + k_1^2}{2t_{\mathrm{S}}L_{\mathrm{C}} + 2t_{\mathrm{S}}r + k_1^2}$$

Converting to massic (specific) activity or volumic activity (activity concentration)

Divide final formulas for R_s, $\sigma(R_s)$, L_C, L_D, and L_U by ε and, m or V

Note: uncertainties due to ε, m, or V were not propagated in this version